Reply to the Comment on "Influence of protons on the capture of electrons by the nuclei of ^7Be in the Sun"

V. B. Belyaev

Bogolyubov Laboratory of Theoretical Physics,

Joint Institute for Nuclear Research, Dubna 141980, Russia

M. Tater and E. Truhlík

Institute of Nuclear Physics ASCR, CZ–250 68 Řež, Czech Republic

Abstract

We show that the arguments against our paper raised by B. Davids *et al.* are either irrelevant or incorrect.

PACS numbers: PACS number(s): 23.40.-s, 25.10.+s, 97.10.Cv

Keywords: electron; capture; continuum; Sun

I. INTRODUCTION

In our reply to the Comment [1] we first summarize shortly results of our work [2] and then we shall analyze the critical remarks of the Comment. Let us note here that we were interested in [2] only in the mechanism of the elementary process of the capture of the electron from the continuum

$$^7Be + e^- + p \rightarrow ^7 Li + \nu_e + p\,, \tag{1.1}$$

and did not discuss the possible plasma effects. In the binary reaction

$$^7Be + e^- \rightarrow ^7 Li + \nu_e\,, \tag{1.2}$$

these effects were found small [3, 4, 5]. Since the mean distance R_0 between protons is about 3×10^4 fm and the Debye radius R_D is about 4×10^4 fm, one can expect that these effects will be small also for the reaction (1.1). Therefore we shall ignore all the discussion of the Comment concerning the description of the plasma.

Necessity to consider the reaction (1.1) follows from the fact that in the standard theory of the *pp* cycle the destruction of the nucleus ^7Be takes place both in the binary reactions (1.2) and

$$p + \, ^7Be \,\rightarrow\, ^8B \,+\, \gamma\,. \tag{1.3}$$

Let us stress that we study [2] the reaction (1.1) quantum-mechanically using the solution of the Schroedinger equation applied in [6] for the description of the system of three charged particles in the continuum. It was shown [6] for the case of two heavy and one light particle that the three-body wave function can be expanded in a small parameter ϵ. In principle, the heavy particles are allowed to interact also strongly. In our case $\epsilon \approx (m_e/m_p)^{(1/2)} \approx 0.0233$. Here m_e (m_p) is the electron (proton) mass. The first term of this expansion is $\Psi_0(\vec{r},\vec{R}) \,=\, \Psi^C_1(\vec{R})\Psi^C(\vec{r}, Z = Z_1 + Z_2)$, where $\Psi^C_1(\vec{R})$ is the Coulomb wave function describing the relative motion of the proton and ^{7}Be and satisfies the equation,

$$-\frac{\hbar^2}{2m_r}\Delta_{\vec{R}}\,\Psi^C_1(\vec{R}) + \frac{Z_1Z_2e^2}{R}\Psi^C_1(\vec{R}) \,=\, E_1\Psi^C_1(\vec{R})\,. \tag{1.4}$$

Here m_r is the reduced mass of the proton-^{7}Be system. In its turn, $\Psi^C(\vec{r}, Z = Z_1 + Z_2)$ is the Coulomb wave function of the electron moving in the continuum in an effective Coulomb potential of the charge $Z = Z_1 + Z_2$ and satisfies the equation,

$$-\frac{\hbar^2}{2m_e}\Delta_{\vec{r}}\,\Psi^C(\vec{r}) - \frac{Ze^2}{r}\Psi^C(\vec{r}) \,=\, E\Psi^C(\vec{r})\,. \tag{1.5}$$

So in contrast to the binary reaction (1.2), in the three-body initial state (1.1) the motion of the electron occurs in the Coulomb field with an effective charge $Z = 5$ and the vector $\vec{r}$ points to the position of the electron relative to the center of mass of the proton-^{7}Be system. It means that to calculate the electron capture by ^{7}Be in the ternary reaction (1.1), one needs to know its wave function $\Psi^C(\vec{r}, Z = 5)$ for the value of $|\vec{r}|$ equal to the distance between the center of mass of the system proton-^{7}Be and the position of ^{7}Be which corresponds to zero distance between the electron and ^{7}Be. This is the main qualitative result of [2] which also contains the quantitative comparison of the effect of the electron density in the ternary state (1.1), given by the function $\Psi^C(\vec{r}, Z = 5)$, with the electron density in the binary reaction (1.2). According to Table I [2], the ratio of these effects is in the Sun of the order of 10 %, but it can be of the order 1 in the dense stars, as can be seen from our Fig. 2 [2].

II. ANALYSIS

Let us now analyze the arguments of the Comment. In our opinion, the content of the Comment contains the three groups of contradictory arguments. To the first group one can relate arguments of the type "three-body mechanism of the electron capture by ^{7}Be is, in fact, a binary one". In the second group of the arguments, the existence of the reaction (1.1) is accepted de bene esse in a sense that if even the three-body mechanism works, both the wave functions and the calculations are in our work totally wrong. To the end of the Comment, Davids *et al.* solve the problem by an argument that the effect of the reaction (1.1) has already been taken into account in [5] by describing the binary reaction (1.2) within the framework of the formalism of the equilibrium plasma and using the Monte Carlo technique to include the interaction of the static protons with the electron and the ^{7}Be nucleus. Non-biased reader can easily follow the evolution of the "proofs" of Davids *et al.* from the full negative of the existence of the three-body effects to the full acceptance of their presence in the equilibrium plasma. We show below that these arguments are either contradictory or have no relation to our work.

Being not able to refute the fact of the absence of works dealing with the explicit three-body mechanism of the capture of the electrons in the continuum by ^{7}Be, Davids *et al.* try to prove that the reaction (1.1) is the binary process. If one omits completely the fundamental difference in the kinematics between the ternary- and binary collisions and also excludes

the influence of the Coulomb interaction of the proton with the electron and ^{7}Be, then the ternary process converts into the binary one. However, these assumptions, admitted by Davids *et al.*, have no relation to the real situation. Following the logics which is difficult to follow, Davids *et al.* compare the mean distance between the particles in the plasma with the range of the weak interaction. Evidently, this comparison has no sense and has no relation to the role of the three-body mechanism of the reaction, because the structure of the weak Hamiltonian has no influence on the formation of the initial state.

The essence of the point of view advocated by Davids *et al.* can be understood from the text presented at the end of the third paragraph at p. 1: *"In fact, the only influence such a proton can have on the electron capture rate is electromagnetic, by affecting the density of electrons at the ^{7}Be nucleus. Therefore it is incorrect to think of this as a ternary reaction. Rather it is a binary reaction in a plasma environment."* This philosophy was implemented in the calculations [5] where the influence of the static protons surrounding ^{7}Be on the capture rate of the electrons in the binary reaction (1.2) was taken into account. It is clear that the static protons do not change the binary feature of the reaction and can cause only the change of the capture rate. But our point of view is that the reaction (1.2) is not the only possible mode of the electron capture and the channel (1.1) also occurs. In this case, the proton possesses the explicit dynamical degree of freedom which means that the Hamiltonian describing the initial state contains not only the Coulomb interaction between the proton, electron and ^{7}Be nucleus, but in addition to the kinetic energy terms of the electron and ^{7}Be also such a term for the proton is present. This changes the situation essentially because instead of only one Jacobi coordinate for the binary electron-^{7}Be system one should introduce two Jacobi coordinates characterizing the three-body proton-electron-^{7}Be system. Then instead of the two-body Schroedinger equation one is to solve the three-body one which was done in [6].

As it follows from the analysis [6] discussed above, the three-body process (1.1) due to the presence of the proton in the vicinity of ^{7}Be, possessing the explicit dynamical degree of freedom, results in the capture of the electron by an effective charge Z=5 instead of Z=4. Moreover, the presence of such a proton in the final state causes that, in contrast to the binary reaction (1.2), the resulting neutrino spectrum is not monoenergetic but continuous one. This is a typical feature of the neutrino spectrum in a reaction resulting in a many-body final state, like the triton beta-decay. It is clear that these features of the ternary reaction (1.1) cannot be simulated by the screening corrections as calculated by the Monte Carlo

simulations in Ref. [5] for the binary reaction (1.2), as wished by Davids *et al.* in the last paragraph of the Comments. Moreover, the claim *"Equilibrium statistical mechanics takes care of the three-body and other effects."* is characteristic, as all the text of the Comments, by the interchange of notions consisting in that we speak about the elementary reaction of the capture whereas Davids *et al.* talk about the description of the plasma effects. However they do not recognize that the argument about the presence of the three-body effects in the equilibrium plasma used against considering independently the three-body elementary process, can be applied also to the binary mechanism of the capture. In other words, following the logics of Davids *et al.*, the binary elementary processes should be also excluded from the treatment which does not make any sense.

Discussing the quality of our wave functions Davids *et al.* argue in the last paragraph on p. 3: *"Its equation 2.3 is a poor approximation to the three body wave function 2.2 in the limit of interest, namely when the electron and the* ^7Be *nucleus are spatially coincident and the proton is some 30 000 fm away from the other two particles. Clearly, this approximation grows worse and worse as the proton-*^7Be *separation* R *increases and the magnitude of the Coulomb wave function describing the relative motion of the proton and* ^7Be *vanishes."* Probably, here Davids *et al.* speak about some other system and have in mind the wave function different from the one given by Eq. (2.3), because the asymptotic of this wave function coincides exactly with the asymptotic of the Coulomb wave function that describes the proton motion in the Coulomb field with the charge Z=3, as it should be in the configuration in which the electron occurs in the vicinity of the ^{7}Be nucleus [6]. Moreover this qualitative reasoning of Davids *et al.* is not supported by any quantitative results. On the contrary, as it is seen from Eq. (2.12) [2], the integration over the variable R excludes the contribution of the wave functions from large and short distances due to the presence of the exponential damping factor. Consequently, possible deformation of the proton-nucleus wave functions due to the screening cannot influence essentially the results.

Let us note here that our wave functions describe correctly not only the $p-e-^7Be$ system, but also the well known $p-e-p$ reaction, providing for the electron function the Fermi function with the effective charge Z=2 and for the pp system the standard quantum mechanical wave function obtained by solving the Schroedinger equation with both the strong and Coulomb interactions included. Namely these wave functions were used to describe the $p-e-p$ reaction by Bahcall and May [7] forty years ago.

In the following text *"The paper asserts that the Coulomb wave function of the electron in the field of the combined charges of the proton and ^7Be, $\Psi^C(\vec{r}, Z = Z_1 + Z_2)$, defines the probability of ^7Be electron capture. In fact, this is the Coulomb wave function describing the relative motion of an electron and ^8B, and is only applicable when the proton is closer to the ^7Be than the electron is. In electron capture this approximation breaks down since the electron-^7Be separation must vanish in order for the capture to occur."*, Davids *et al.* show clearly that they missed completely the essence of our work and try to palm something off on us which is complete nonsense and what has nothing to do with what we have done. Let us stress once more that the wave function $\Psi^C(\vec{r}, Z = Z_1 + Z_2)$ does describe the motion of the electron in the effective field of the proton-^7Be system with $Z = Z_1 + Z_2 = 5$, and the vector $\vec{r}$ does point from the center of mass of the proton-^{7}Be system to the electron. It follows that the value of the electron coordinate $|\vec{r}| = R/7$ is the distance of ^{7}Be to the center of mass of the p-^{7}Be system. As to the act of the electron capture by ^{7}Be, our picture is : The modulus squared $|\Psi^C(r = R/7, Z = 5)|^2$ provides the probability that the electron can be registered at the point $|\vec{r}| = R/7$ where ^{7}Be is situated and can capture the electron. Since the expansion (2.2) and Eq. (2.3) [2] are valid in the whole space of the variables $\vec{r}$, $\vec{R}$, we conclude here that the above quoted reasoning of Davids *et al.* is just a bleak fiction.

In summary, we have shown that the arguments by Davids *et al.* that the reaction (1.1) can take place in the solar plasma only with the proton as a spectator the influence of which has already been taken into account in [5] by calculating the screening to the binary reaction (1.2) are false. On the other hand we are aware that our model calculations [2] cannot be considered as a substitute for full calculation of this process that only can provide the reliable information on the size of its rate.

---

[1] B. Davids, A.V. Gruzinov and B.K. Jennings, arXiv: 0705.4669.

[2] V.B. Belyaev, M. Tater and E. Truhlík, Phys. Rev. **C 75**, 034608 (2007).

[3] J.N. Bahcall and C.P. Moeller, Astrophys. J. **155**, 511 (1969).

[4] C.W. Johnson, E. Kolbe, S.E. Koonin and K. Langanke Astrophys. J. **392**, 320 (1992).

[5] A.V. Gruzinov, J.N. Bahcall, Astrophys. J. **490**, 437 (1997).

[6] V.B. Belyaev, S.B. Levin and S.L. Yakovlev, J. Phys. B: At. Mol. Opt. Phys. **37**, 1369 (2004);

physics/0310105.

[7] J.N. Bahcall and R.M. May, Astrophys. J. **155**, 501 (1969).